和歌山 数学 [illegible]

$$\lim_{x\to 0}\frac{x^3}{\log\frac{1-x}{1+x}+2x}=\lim_{x\to 0}\frac{3x^2}{\frac{1+x}{1-x}\cdot\frac{-(1+x)-(1-x)}{(1+x)^2}+2}$$

$$=\lim_{x\to 0}\frac{3x^2}{\frac{1+x}{1-x}\cdot\frac{-1-x-1+x}{(1+x)^2}+2}$$

$$=\lim_{x\to 0}\frac{3x^2}{\frac{-2}{(1-x)(1+x)}+2}$$

$$=\lim_{x\to 0}\frac{3x^2(1-x)(1+x)}{-2+2(1-x)(1+x)}$$

$$=\lim_{x\to 0}\frac{3x^2(1-x^2)}{-2+2(1-x^2)}$$

$$=\lim_{x\to 0}\frac{3x^2-3x^4}{-2+2-2x^2}$$

$$=\lim_{x\to 0}\frac{6x-12x^3}{-4x}$$

$$=\lim_{x\to 0}\frac{6-36x^2}{-4}$$

$$=-\lim_{x\to 0}\frac{6}{4}+\lim_{x\to 0}8x^2=-\frac{3}{2}$$

$$\frac{d}{dx}\left(\frac{1-x}{1+x}\right)$$

$$=\frac{-(1+x)-(1-x)}{(1+x)^2}$$

$$=\frac{-1-x-1+x}{(1+x)^2}$$

$$=-\frac{2}{(1+x)^2}$$

変数変換すると x, y は u, v をもちいて

①+②　　　　①−②

$u+v=2x$　　　　$u-v=2y$

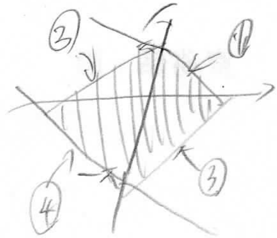
$x=\dfrac{u+v}{2}$　　　　$y=\dfrac{u-v}{2}$

となる。これから領域Dは

$D=\{(u,v) \mid -\pi \le u \le \pi,\ -\pi \le v \le \pi\}$

となる。ヤコビアンJは

$$J=\left\|\begin{matrix}\dfrac{\partial x}{\partial u} & \dfrac{\partial x}{\partial v}\\ \dfrac{\partial y}{\partial u} & \dfrac{\partial y}{\partial v}\end{matrix}\right\| = \left\|\begin{matrix}\dfrac{1}{2} & \dfrac{1}{2}\\ \dfrac{1}{2} & -\dfrac{1}{2}\end{matrix}\right\| = \left|\dfrac{1}{2}\cdot\left(-\dfrac{1}{2}\right)-\dfrac{1}{2}\cdot\dfrac{1}{2}\right| = \dfrac{1}{2}$$



$|y| \le \pi - |x|$

$x+y \le \pi$

$|y| \le \pi -$

① $y \le \pi - x$

② $y \le \pi + x$

③ $-y \le \pi - x$

$y \ge x -$

④ $-y \le \pi +$

$y \ge -x -$

より

$$\iint_D (x+y)^2 \cos\frac{x-y}{2}\,dx\,dy = \int_{-\pi}^{\pi}\left\{\int_{-\pi}^{\pi}\frac{1}{2}u^2\cos\frac{v}{2}\,du\right\}dv$$

$$=\frac{1}{2}\int_{-\pi}^{\pi}u^2\,du\int_{-\pi}^{\pi}\cos\frac{v}{2}\,dv$$

$$=2\int_0^{\pi}u^2\,du\int_0^{\pi}\cos\frac{v}{2}\,dv$$

$$=2\left[\frac{1}{3}u^3\right]_0^{\pi}\left[+2\sin\frac{v}{2}\right]_0^{\pi}$$

$$=\frac{2\pi^3}{3}\left(+2\sin\frac{\pi}{2}\right)=\frac{4}{3}\pi^3$$

$$\int_0^{\pi}\cos\frac{v}{2}\,dv$$

$$=\int_0^{\pi}\frac{1+2\cos v}{2}\,dv$$

$$=\frac{1}{2}\left[v+2\sin v\right]_0^{\pi}$$

$$=\frac{1}{2}(\pi+2\sin\pi)$$

$$=\frac{\pi}{2}$$

$$\left|\begin{pmatrix} 0 & x & 0 \\ 0 & 0 & x \end{pmatrix} - \begin{pmatrix} c & a & b \\ a & b & c \end{pmatrix}\right|$$

$$= \begin{vmatrix} x-b & -c & -a \\ -c & x-a & -b \\ -a & -b & x-c \end{vmatrix} = (x-a-b-c) \begin{vmatrix} 1 & -c & -a \\ 1 & x-a & -b \\ 1 & -b & x-c \end{vmatrix} =$$

$$= (x-a-b-c) \begin{vmatrix} 1 & - \\ 0 & - \\ 0 & - \end{vmatrix}$$

$$= (x-a-b-c) \begin{vmatrix} -x-a+ \\ -b+c \end{vmatrix}$$

$$= (x-a-b-c)((x-a+c$$
$$-(-b+a)(-b+c)$$
$$= (x-a-b-c)((x^2-(a-$$
$$-(b-a)(b-c))$$
$$= (x-a-b-c)(x^2-(a-c$$
$$-(b^2-(a+c)b+$$

$$B = \begin{pmatrix} c & a & b \\ a & b & c \\ b & c & a \end{pmatrix} \to \begin{pmatrix} c & a & b \\ b & c & a \\ a & b & c \end{pmatrix} \to \begin{pmatrix} b & c & a \\ c & a & b \\ a & b & c \end{pmatrix} = A$$

Bの固有多項式はAの固有多項式と同じである。

$=b=c$のときのAは

$$A = \begin{pmatrix} a & a & a \\ a & a & a \\ a & a & a \end{pmatrix}$$

固有多項式はxと定数a

$$|xE-A| = \left|\begin{pmatrix} x & 0 & 0 \\ 0 & x & 0 \\ 0 & 0 & x \end{pmatrix} - \begin{pmatrix} a & a & a \\ a & a & a \\ a & a & a \end{pmatrix}\right|$$

$$= \begin{vmatrix} x-a & -a & -a \\ -a & x-a & -a \\ -a & -a & x-a \end{vmatrix} = 0$$

$$(x-3a) \begin{vmatrix} 1 & -a & -a \\ 1 & x-a & -a \\ 1 & -a & x-a \end{vmatrix} = 0$$

$$(x-3a) \begin{vmatrix} 1 & -a & -a \\ 0 & x & 0 \\ 0 & 0 & x \end{vmatrix} = 0$$

$$x^2(x-3a) = 0$$

$$x = 3a, 0 \text{(重解)}$$

のときの固有ベクトル $v_1 = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$ は

$$1 = 3a v_1$$

$$aE)v_1 = 0$$

$$\begin{pmatrix} a & a \\ -2a & a \\ a & -2a \end{pmatrix}\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} 0 \\ 0 \\ 0 \end{pmatrix}$$

$$\left(\begin{array}{ccc|c} -2a & a & a & 0 \\ a & -2a & a & 0 \\ a & a & -2a & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -2 & 1 & 1 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -2 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -1 & -1 & 2 & 0 \\ 1 & -2 & 1 & 0 \\ 1 & 1 & -2 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -1 & -1 & 2 & 0 \\ 1 & -2 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -1 & -1 & 2 & 0 \\ 0 & -3 & 3 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$\left(\begin{array}{ccc|c} -1 & 0 & 1 & 0 \\ 0 & -1 & 1 & 0 \\ 0 & 0 & 0 & 0 \end{array}\right)$$

$$-x_1 + z_1 = 0$$

$$-y_1 + z_1 = 0$$

$$\begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix} = \begin{pmatrix} z_1 \\ z_1 \\ z_1 \end{pmatrix} = k\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}$$

$y' = -y$

$\int \frac{1}{y^2} dy = -\int \frac{1}{x^3} dx$

$-\frac{1}{y} = +\frac{1}{2x^2} + C_1$

$y = \frac{-1}{\frac{1}{2x^2} + C_1}$ 但C, C_1は任意定数

$xy'' = 1$

$y'' = \frac{1}{x}$

$y' = \int \frac{1}{x} dx$

$= \log|x| + C_3$

$y = \int (\log|x| + C_3) dx$

$= x\log|x| - \int dx + C_3 x + C_4$

$= x\log|x| - x + C_3 x + C_5$

$= x(\log|x| - 1 + C_3) + C_5$

$= x(\log|x| + C_6) + C_5$

但C, C_3, C_4, C_5, C_6は任意定数

$\int x^n = \frac{1}{n+1} x^{n+1}$

$y = \frac{1}{-\frac{2}{x^3} + C_2}$

$= \frac{x^3}{-2 + C_2 x^3}$

$y' = \frac{3x^2(-2 + C_2 x^3) - \dots}{(-2 + C_2 x^3)\dots}$

$x^3 y' = \frac{3\frac{-2 + C_2 x^3}{x} - 3C\dots}{(-2 + C_2 x^3)\dots}$

$+ \frac{x^6}{(-2 + C_2 x^3)}$

$y' = \frac{1}{(\frac{1}{2x^3} + C_1)^2} \cdot -\frac{\ }{x^3}$

$x^3 y' = -\frac{\cancel{x}\,?}{(\frac{2}{x^3} + C_1)^2}$

$y^2 = \frac{1}{(\frac{2}{x^3} + C_1)^2}$

$y' = \log|x| + C_6 + 1$

$y'' = \frac{1}{x}$

$= \frac{y}{x} + \sqrt{1+\frac{y^2}{x^2}}$

$\frac{y}{x}$ とおくと

$y = ux$

$\frac{dy}{dx} = x\frac{du}{dx} + u$

$x\frac{du}{dx} + u = u + \sqrt{1+u^2}$

$x\frac{du}{dx} = \sqrt{1+u^2}$

$\int \frac{1}{\sqrt{1+u^2}} du = \int \frac{1}{x} dx$

$\int \frac{1}{\sqrt{1-u^2}} du = \sin$

において $t = u + \sqrt{1+u^2}$ とおくと

$\frac{dt}{du} = 1 + \frac{2u}{2\sqrt{1+u^2}}$

$= 1 + \frac{u}{\sqrt{1+u^2}}$

$\frac{1}{1+\frac{u}{\sqrt{1+u^2}}} dt = du$

$\frac{\sqrt{1+u^2}}{u+\sqrt{1+u^2}} dt = du$

$\frac{1}{t} dt = \frac{1}{\sqrt{1+u^2}} du$

$\int \frac{1}{t} dt = \int \frac{1}{x} dx$

$\log|t| = \log|x| + C_7$

$\log\left|\frac{t}{x}\right| = C_7$

$\frac{t}{x} = \pm e^{C_7}$

$C_8 = \pm e^{C_7}$ とおいて

$\frac{t}{x} = C_8$

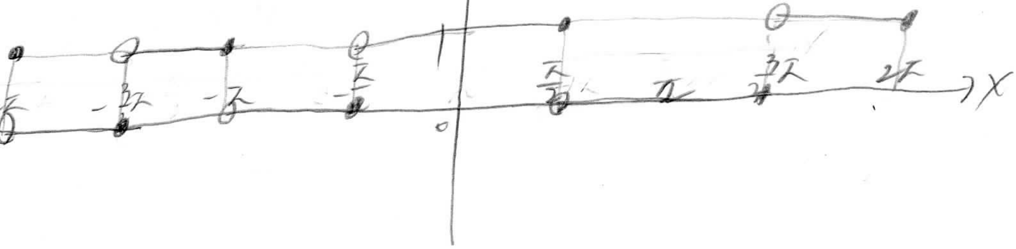
$t = C_8 x$

$u + \sqrt{1+u^2} = C_8 x$

$\frac{y}{x} + \sqrt{1+\frac{y^2}{x^2}} = C_8 x$

$y + \sqrt{x^2+y^2} = C_8 x^2$

但 C_1, C_7, C_8 は任意定数



(1)より $f(x)$は偶関数なので
フーリエ係数 a_0, a_n, b_n のうち

$$b_n = 0$$

a_0, a_n は周期 $2L$ として

$$a_0 = \frac{1}{L}\int_{-L}^{L} f(x)\,dx$$

$$= \frac{1}{\pi}\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}} dx$$

$$= \frac{2}{\pi}\int_{0}^{\frac{\pi}{2}} dx$$

$$= \frac{2}{\pi}\cdot\frac{\pi}{2} = 1$$

$$a_n = \frac{1}{L}\int_{-L}^{L} f(x)\cos\frac{n\pi x}{L}\,dx$$

$$= \frac{2}{\pi}\int_{0}^{\frac{\pi}{2}} \cos nx\,dx$$

$$= \frac{2}{\pi}\left[\frac{1}{n}\sin nx\right]_0^{\frac{\pi}{2}}$$

$$= \frac{2}{n\pi}\left(\sin\frac{n\pi}{2}\right)$$

$$= \begin{cases} 0 & (n=2k) \\ \frac{2}{(2k-1)\pi}(-1)^{k+1} & (n=2k-1) \end{cases} \quad (k=1,2,\cdots)$$

n	k	
1	1	$-$
2		0
3	2	$+$
4		0

$f(x)$のフーリエ級数は

$$f(x) \sim \frac{1}{2} + \frac{2}{\pi}\sum_{k=1}^{\infty}\frac{(-1)^{k+1}}{2k-1}\cos nx$$